Design and Analysis of Single-Phase Power Transformers for Undergraduate Engineering Students

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Abstract

This paper describes a method for design optimization of single-phase power transformers using an interactive PC-based computer program. The computer program is developed in house and in close cooperation with industrial users. A procedure is developed to illustrate the effect of parameter variation on the design of transformers in order to achieve minimum cost of production. The procedure illustrates that there are many possible designs within a very small increment of cost. The objective is to assist undergraduate students to understand the design process: determining the efficiency, size, weight and cost of actual transformers, while meeting multiple transformer specifications.

1. Introduction

Most technical papers on computer-aided design of power transformers utilize optimization routines which guide the choice of the independent variables to optimize a design [1-3]. Thus, the computer program regulates the design skill and isolates the student from the design process. In these approaches, the initial estimates are introduced in the computer and the computer automatically achieves a design which meets all specifications, regardless of any error in the initial estimates. In this manner, the judgment of the student does not affect the success and the quality of the design, but only the computer time and expense. A testing experiment is described in a fourth paper [4], in which students design, build and test a simple single-phase transformer that used in many textbooks to specifications provided by the instructor. However, it does not appear to be a design experience in the classical design principles. In addition, students cannot actually simulate and analyze the performance of the design.

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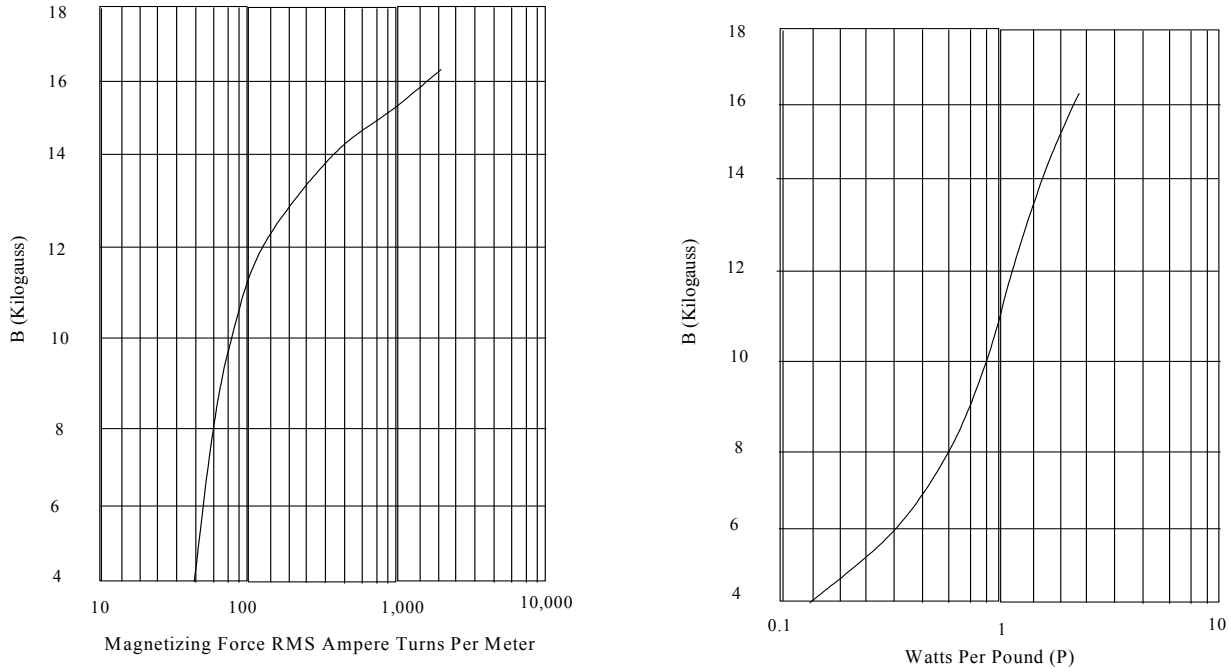
This paper demonstrates the use of a novel PC-based interactive computer program that has been written specifically for the design and analysis of single-phase transformers. The program and user manual is developed in house and in close cooperation with industrial users. The paper takes the student, step by step, through the understanding of transformer designs. The approach followed in this paper is to use the computer for the tasks it does best: storage and retrieval of data, also making routine calculations, but leaving the judgment to the students. It is our intention to keep the real skill and the decision making resident with the students themselves. The computerized interactive approach is used basically for the following rationale: 1) to minimize designing time, 2) its accuracy and capabilities, and 3) it represents state-of-the-art engineering. The approach is surprisingly simple and efficient.

2.0 Typical Design Information

During the first design session the transformer specifications are presented.

- Apparent power rating: 25KVA
- Primary voltage rating V_p: 2500V
- Secondary voltage rating V_s: 250V
- Excitation current of the transformer cannot exceed 2% of the full load current
- Copper to core loss ratio must remain within 1.2 and 2.69
- Maximum efficiency occurs between 75% and 100% of the full load

A magnetic data for a typical 60Hz power transformer is also provided. A core magnetization curve is shown in Fig. 1 [5].



(a) Magnetizing force (b) Core loss

Fig. 1 Core magnetization curve

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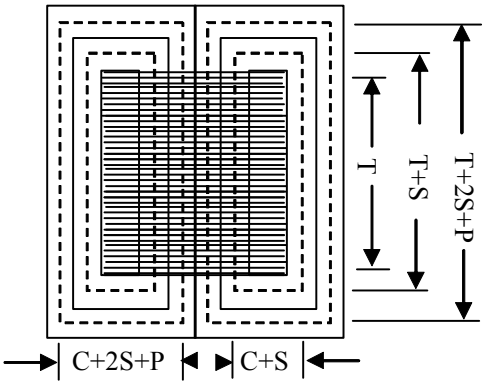
The following information about the transformer is provided to the students as well [5]:

- A typical stacking factor is about 0.95
- The cost of most mass produced equipment is proportional to the equipment weight or mass. A reasonable estimate of this cost is about 4.4 dollars/kg.
- A typical current density in transformer winding is about two amperes per square millimeter.
- In designing the coils, an allowance must be made for the space between the conductors and for the thickness of the insulation on the conductors. In typical transformers, the conductors themselves will only occupy about half of the available window area. Thus, a typical space factor is about 0.5.

The PC-based program is intended for designing the following classes of transformers:

- Single-phase, core type
- Single phase, shell type

The core and coils of the two transformer types modeled by the program are shown in Figs. 2 and 3, respectively. As with any design procedure, a number of assumptions are made.



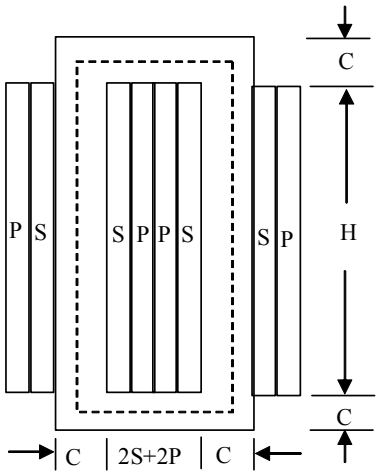


Fig. 2 Geometry of a core type transformer

In designing both transformers, the following assumptions are made:

- The cores have been proportioned so that the flux density is uniform throughout the core
- The low-voltage coil has been wound closest to the core
- The high-voltage coil has been wound over the top of the low-voltage coil
- The number of turns are rounded to the nearest integer
- Temperature is uniform. The transformer operates at 60 degrees centigrade
- The cost of non-metals is ignored

3.0 Program Input

The instructor provided students with an outline of the program that is available for use. The program is designed so that the user will only have to operate in one screen. When the program is accessed, the user is immediately asked to input the first of twenty-one data inputs, and these inputs are manipulated to achieve the stated objectives. The inputs are divided into three sections, particularly:

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- Rated values, such as, the primary and secondary voltages, transformer rating, and frequency
- A set of constants, in this case, the current density, space factor, stacking factor, wire-cross-sectional areas, resistance of the primary and secondary conductors, and the copper and iron densities.
- A set of independent variables, such as, core thickness, width of core leg, window height, and peak value of flux density.

The inputs from one to ten either specify the rating of the transformer or are constants. Inputs 11, 12, and 13 specify the magnetic condition of the core. A reasonable value of flux density is chosen by the student just below the saturation region and the corresponding values of the field intensity and core loss factor are obtained from the magnetization curve. Inputs 14 through 18 specify the coil constants. Once the current density has been specified, the wire size and the resistance can be obtained from wire tables [6, 7]. Inputs 19, 20 and 21 are the basic core dimensions that are needed to interrelate with the various magnetic and electrical quantities.

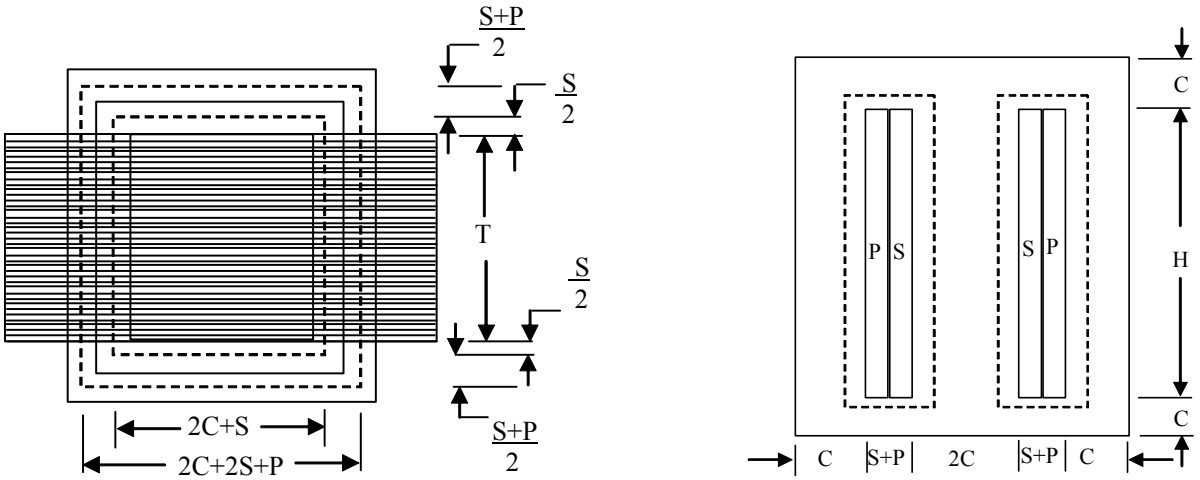

Fig. 3 Geometry of a shell type transformer

4.0 Cost Function

Since the transformer design is intended for educational purpose, the following cost functional is specified as the total cost of the transformer:

Total transformer cost ($) = (Total transformer mass (kg))*(Cost factor ($/kg))

where:

Total transformer mass (kg) = (Total copper mass (kg)) + (Mass of iron (kg))

The optimum design is the one with the minimum value of the cost function.

5.0 Students Preparatory Work

All program variables are converted to the International Systems (SI) of units to ensure consistency in the design. The transformer ratings and the primary and secondary voltages are used to calculate the primary and secondary currents respectively. These determined values of the primary and secondary currents are used in conjunction with the current density to calculate the cross-sectional area of primary and the secondary conductor correspondingly. Since all

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conductors are manufactured to meet international standards with respect to cross-sectional area (wire gauge), the calculated areas are used to select standard wire size for the primary and secondary windings from a wire table. Table I [6], provided the resistance based on the cross-sectional area at 20 degrees C. For the selected wire gauge, which best satisfied the design specifications for the primary conductor; the corresponding resistance per unit length at the given temperature is also noted. Since, this value is not at the required temperature for the design; students extrapolated it to 60^0C. A similar procedure is used to determine the resistance per unit length of the secondary conductor at 60^0C.

One of the most important factors in transformer design is the resistance per meter of the primary and secondary wires at the specified design temperature. The resistance per meter at the design temperature allows the designer to choose the wire gauges that are capable of carrying the primary and secondary currents. If the resistance per meter is known, the corresponding wire gauge can be found from a wire table. Unfortunately, in many cases the resistance per meter in the wire table may have been calculated at a different temperature to the temperature that is required in the design. This problem is encountered in this transformer design. Therefore, the resistance per meter had to be extrapolated to obtain the corresponding value at 60 degrees. Consequently, the primary coils of the optimal transformer, which have an area of $5.0*10^{-6}$ m^2, are designed using American Wire Gauge (AWG) #1/0. This gauge number actually corresponds to a primary wire of cross sectional area $5.26*10^{-6}$. Since the exact wire gauge corresponding to the primary wire with the above cross sectional area could not be found, wire gauge #10 was used for the primary coils. The wire gauge number for the secondary coils is chosen in a similar manner. The secondary coils are designed using AWG #1/0. This gauge number actually corresponds to a secondary wire with cross sectional area of $5.35*10^{-5}$ m^2. Since the cross sectional area of $5.35*10^5$ m^2 is extremely close to the area of the secondary wire, AWG #10 is used to design the secondary coils. In addition, the magnetic field intensity and the core loss factor are determined for each increment of desired flux density before design alternatives are initiated. For a selected flux density, the corresponding magnetic field intensity and core loss factor are obtained from the magnetizing curve and the core loss curve respectively. The flux density and magnetic field intensity are both found from the magnetization curve, and the core loss factor corresponding to the chosen flux density is obtained from the core loss curve. The following equation is used to obtain the extrapolation results at 60 degrees [7].

$R = R_o[1+\alpha(T-T_o)]$

where, R_o is the resistance per meter at reference temperature of 20^o C, α is the temperature coefficient of copper $= 3.9*10^{-3}/^o$ C, and T_o, is the reference temperature.

The weigh of the transformer is a function of the dimensions chosen for the core thickness, core leg width and window height. These dimensions determined the volume and ultimately the mass of iron needed for the core. Also, the dimensions coupled with the number of turns and wire size determined the copper mass needed for a particular design. The cost of the transformer is derived from the actual weight as this is factored into the software as a constant at 4.4$/kg. That is, weight and cost are directly related such that an inexpensive design is one that has a relatively low mass.

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Table I Wire table at 20 degree C

AWG B&S Gauge	Maximum Resistance ohms/1000ft	ohms/m at 20 °C	Square meters	
4/0	0.05045	0.000165	0.000107226	
3/0	0.06361	0.000209	8.50321E-05	
2/0	0.08021	0.000263	6.74192E-05	
1/0	**0.1011**	**0.000332**	**5.34773E-05**	**Secondary**
2	0.1625	0.000533	3.36322E-05	
4	0.2584	0.000848	2.11483E-05	
6	0.4108	0.001347	1.33032E-05	
8	0.6533	0.002143	8.36773E-06	
10	**1.039**	**0.003408**	**5.26128E-06**	**Primary**
12	1.652	0.005419	3.30903E-06	

6.0 Computer-Student Interaction

The class is divided into teams to endorse team-work. Each team is given the same material costs for known materials but each team uses distinct transformer type modeled by the program. Primarily, the students examine the input requirements of the computer program and determine how they will obtain an initial set of input variables. Students also study the trade-off relationships that exist between the various inputs and outputs. The program serves as an aid to the students, allowing them to dispose of the numerous and tedious calculations in a short period of time. The program variables, representing the basic geometry of the core, in addition to the rating data and the electrical specifications are initialized with feasible input values by the students. With this information a complete set of performance calculations is made. These calculations include the cross-sectional area of the core and both the high and low voltage coils, the mean length of the flux path, peak flux and RMS value of the magneto-motive force, the core losses, the mean length of a turn, the total length, the resistance, the copper losses, the number of turns for both high and low voltage coils, the volume and mass of both the steel core and the copper windings of the transformers, the overall core dimensions, the copper to core loss ratio, the KVA to kg ratio, efficiency, the excitation current and the estimated cost of the design. The students then use these values to determine if the performance requirements for the transformer design have been met. In most cases the calculated performance will not agree with the specifications and it will be necessary to modify the initial estimates. This modification of estimates is not a part of the computer program and is left to the judgment of the students. Thus, the judgment of the students does affect the success and the quality of the design. When any of the performance specifications, supplied by the students, are not met, students adjust the appropriate input data and re-executing the program. This process continued until any one of the following occurs: 1) all input specifications are met, 2) the performance specifications are deemed impossible to achieve with the physical characteristics of the design, or 3) the number of iterations reaches twenty.

Even though the program aided the students in the design process, it did not optimize their design alternatives as the output files only show the practical feasibility of the inputted parameters. As a result, the students had to analyze and interpret the output of the program for each design alternative and then developed a systematic procedure to optimize their design. Initially, Students spent hours using a rugged and brute force method to choose the independent

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variables, as the output from the program is a reflection of their chosen inputs. Consequently, for a particular design alternative, the computer program could not manipulate their inputs to give a better design. Instead, careful analysis of the output files for different inputs revealed trends, which are used to develop a systematic or a more logical approach towards arriving at the best design. After progressing thus far, they acknowledged the fact that the program has made their design process simpler as the data it generated would have consumed several more hours of their time if they had calculated it manually for each design alternative. Accordingly, it is on this level of interaction with the program that students strategize input files, analyze and interpret the output. Then further manipulate one or two variables while keeping others fixed to optimize the design. In this way the design is successively built upon, or synthesized by the student until a final, satisfactory design is achieved. After completing all the calculations, the students realized that there are only four independent variables, which they could have manipulated to obtain the best design. These independent variables were flux density, B, the core thickness, T, core leg width, C, and the window height, H. All four variables affected specifications, which are critical in meeting the design criteria.

7.0 Design Optimization of Core-Type Transformer

7.1 Initial Design

The initial design is the product of a brute-force method. Values for the independent variables were arbitrarily chosen and the practicality of these random designs, are analyzed for trends. The independent variables are then incremented in a controlled and systematic manner to improve the performance characteristics of the transformer. Students realized from the B-H curve that the possible choices of the flux density are within the range of 12 kilogauss (just above the knee of the curve) to 14 kilogauss, so increments of the flux density are read off from the curve. In the preliminary stages of the design, a flux density, B=1.12 Tesla and the corresponding values for the magnetic field intensity, and core loss factor are determined from the B-H curve. Next values for T = 0.5 m, C = 0.25 m, and H = 0.075 are arbitrarily chosen to complete their initial design. These values are used in conjunction with the other predetermined constants and rated values, after which the design is simulated. The output is observed and recorded for future reference. At this time, students are unable to meet any of the design specifications. The efficiency is just over 90% and the copper to core loss ratio is 3.34. In addition, the cost of this design is almost $5000.00. From these results, students recognized that the core leg width and the thickness of the transformer are too high. By keeping the flux density fixed, this brute force approach is applied in choosing other values for T, C and H. In addition to holding the flux density fixed, one of the other independent variables is also held constant routinely while varying the other two. This is done to approximate the best range for the core thickness, core leg width and window height. Instantaneously students had acquired a profound knowledge or the trade off relationships among the design parameters so they pooled all the observed trends together, in an effort to increase the efficiency of the transformer.

In order to illustrate the impact of the flux density on the design process, student chosen new values for C= 0.05m, T = 0.135m, and H=0.07m, as the new benchmark or reference point by holding them constant and then vary the flux density. As the flux density is increased from the range of 1.2 to 1.4, the cost of the transformer decreased. Interestingly, the efficiency at the

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rated load remained almost constant as for every 0.025 increment of the flux density, the efficiency decreased by less than 0.001 percent. Even though it is favorable for the transformer weight and cost to decrease as flux density is increased, the excitation current increased with flux density while the copper to core loss ratio decreased. This further increased the complexity of the design. That is, in order to meet the design specifications, there must be a trade off between flux density and cost.

8.0 Design Alternatives

Since the design constraints such as are within the specified limits, students narrowed their selection down to the economics of the various design alternatives. The optimum design is obtained after meticulously analyzing the output data. A number of design alternatives are produced to judge the performance of the transformer design. However, for brevity, only the following four design alternatives are reported in this paper with a peak flux density of 1.3 Tesla, and are given in Figs. 4-7

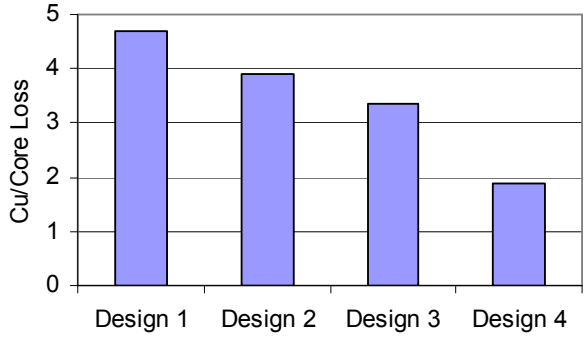

Fig.4 Comparison of cu/core loss ratio

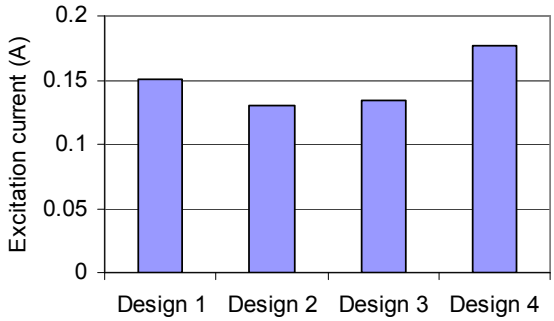

Fig.5 Comparison of excitation current

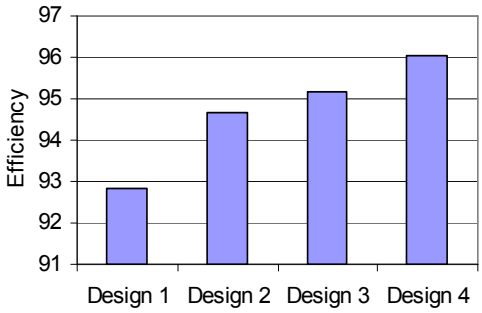

Fig. 6 Comparison of transformer efficiency

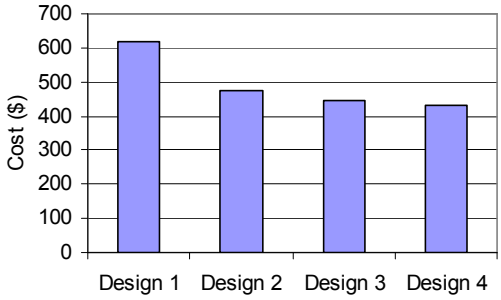

Fig. 7 Cost comparison

In design alternative 1, values for T=0.135m, C=0.05m, and H=0.08m produced a satisfactory excitation current, but the copper to core loss ratio is too excessive, and there is an increase in the transformer cost as well. This design satisfied the excitation current requirement, but the copper to core loss ratio is too large and exceeds the specifications. In order to find a new set of independent variables for a second design alternative, students chosen T=0.135m, C= 0.05m, and H=0.15m for alternative 2. A closer inspection of this alternative reveals that the

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copper to core loss ratio is not even close to the desired range and burdensome weight for the transformer. A third alternative with T=0.135m, C= 0.05m, and H=0.2m, produced a transformer with a full rated load efficiency of 95.1509. This design satisfies the excitation current, but copper to core loss ratio is too large, and there is increase in the transformer weight. However, alternative 3 shows a better efficiency over alternative 2and a cost of about $25.0 less than alternative 2. Design alternative 4 is a good quality design that meets all the specified criteria with T=0.15 m, C=0.06m, and H=0.2m. Thus, this transformer provides an optimum design with a full rated efficiency of 96.0314%, a copper to core loss ratio of 1.89596, which fell within the specified range of 1.2 to 2.69. The transformer's excitation current of 0.176077A also remained below the 0.2A maximum limit set in the design specification. Improved performance is provided in a compact 44.242 kg transformer at a manufacturer cost of $433.7.

9.0 Design of a shell-Type Transformer

For the shell design, students decided to try using the optimum core design as an initial starting point (alternative 1). They are able to tweak the dimensions of the shell-type transformer to obtain the right ration for the optimum design. The following design alternatives are obtained and are displayed in Figs. 8-11. The optimum design is obtained from the above three transformers. The selected optimal design is quite successful in meeting all of the design specifications

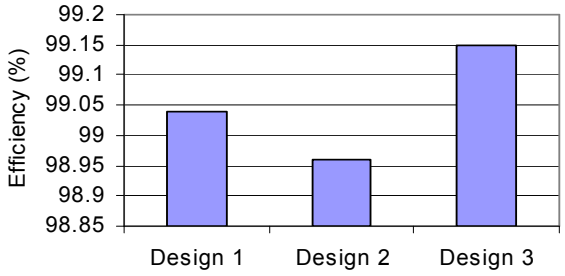

Fig. 8 Comparison of efficiency

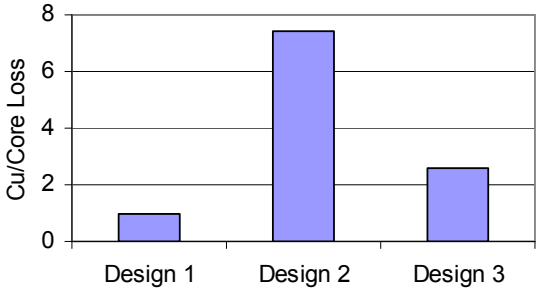

Fig.9 Comparison of copper/core loss ratio

In alternative 1, students changed the dimensions of the optimum core design as follows: 1) core thickness from 0.15m to 0.239m, 2) the width from 0.06m to 0.089m, and 3) the window height from 0.2m to 0.354m. They observed that for a significant increase in the efficiency of the transformer, there are noticeable changes in the excitation current which tends toward the critical value of its design. Similarly, this upward trend of the excitation current is complemented by an increase in the overall transformer weight and ultimately the cost. Thus, alternative 1 is worthless. Values for T=0.153m, C= 0.057m, and H=0.267m are chosen for a second design alternative, while T=0.191m, C= 0.0713m, and H=0.283m are selected for a third design alternative. Alternative 2 shows a slightly better weight over alternative 3 and also a cost of about $15.3 less than alternative 3, but the copper to core loss ratio is enormous and surpasses the transformer specifications. As a result, this transformer is omitted in favor of alternative 3. Clearly, alternative 3 provides an optimum design with a full rated efficiency of 99.15, a copper to core loss ratio of 2.603, and a cost of $430.5.

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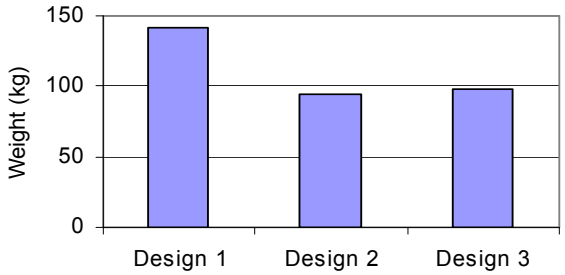

Fig. 10 Weight Comparison

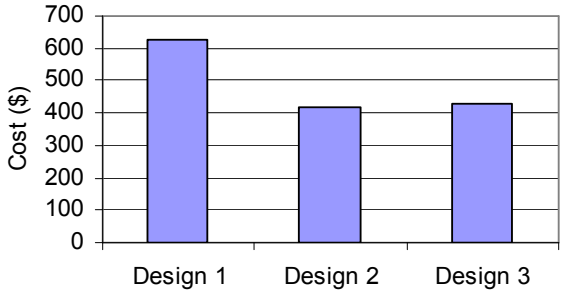

Fig. 11 Cost comparison

10.0 Conclusions

This paper described a method to design a single-phase transformer. A set of design steps has been built-up to influence a set of independent variables in order to attain the most economical design by minimizing losses and weight, while meeting performance specifications. The critical relationship between flux density and cost was analyzed. It is depicted that for a chosen value of the flux density, there are many designs, which satisfied the design criteria but there exists unique values for the core thickness, core leg width and window height, which yielded minimum cost. Students optimized their design by determining how close they have come to meeting the transformer specifications. Upon completion of the design, the students have been through the entire design process, from initial design, through redesign, and analysis. They have dealt with multiple specifications that can conflict, such as balancing efficiency, copper to core loss ratio, and exciting current. They have seen a number of trade-offs that can be made in the design process. The procedure was completed by using the decision making process to rationally select the design which best satisfied the criteria.

11.0 Program Availability

The program utilized in this paper is available to Electrical Engineering Educators at no cost. Interested individuals may contact the lead author at arubaai@howard.edu.

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